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***EXPRESSIVITY***  
**AS A CONDITION OF  
EXPRESSION**

AKSIOMA

## **Expressivity as a Condition of Expression**

Suppose we analytically divide conditions of expression into formal and material conditions. Roughly the material conditions are about having something to express at all and the formal a kind of limit that circumscribes the set of expressible things. When talking about expressivity as a condition of expression we are then not directly interested in the usual questions about freedom of expression as in not being censored and related important issues in communication and interpretation as in having expressions be understood by others. It is closer to questions regarding the thinkable, the sayable and the relations between them.

Expressivity deals with questions relating to what is or is not expressible in a system. This becomes quite tricky when it comes to dealing with natural languages which are arguably the main medium for expression. It would lead us deep into philosophy of language and such questions as the normativity of grammar. So this essay will only explore a couple of episodes in different more or less formal systems in order to build the reader's intuition about expressivity, its history and perhaps its future. When it comes to artificial languages and other formal systems the question of expressivity can appear as the question of the range of possible expressible notions within this system. This is the scale on which history offers useful examples for a primer on expressivity. However, since artificial languages and formal systems are not only (and sometimes not at all) means of communication, this analogy with natural languages leads us to the realm of technology.

The motivation for learning about expressivity and its history in this way is at least threefold. First, it's interesting to see how at certain points a limit of expressivity was reached and then overcome. These moments are often related to some of the major advances

in science and mathematics or are perhaps even their condition. Second, for some systems there are interesting proofs about limits of certain formal systems, these proofs are often themselves landmark results and the techniques they entail can be used in analogous settings. And third, we can try to reason about the feedback flows of these changes within these systems into the changes in the expressivity of natural language.

The opposite flow, that is the question of the movement of ideas from natural language into formal systems is quite clearly rendered in the framework of the idea of explication put forward by Rudolph Carnap.<sup>1</sup>

$$S \xrightarrow{E} F \times I$$

An explication  $E$  is a transformation taking a (vague) source concept  $S$  into a pair of a formalism  $F$  with an interpretation  $I$ .

Explication here is a process of transforming a (more or less) informal concept, usually taken from natural language into a more formal setting. A familiar example of this situation is quantification; which is usually theorized in a critical register within this pattern: by quantification, the richness of the intuitive concept seems to be reduced and it seems something important is being lost. For example, this kind of critique is often targeted against IQ, where it is common to complain that the measure reduces the meaning of intelligence and leaves important parts of its meaning out. It can and often is argued that this type of critique is valid since the price of precision gained by the quantification does not pay off epistemically or because this

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1 The theory presented here is an elaboration of the one developed in Carnap's book the logical foundations of probability (Carnap 1971).

reduction is causing unjustified (social) damage. Looking through the prism of expressivity, more sweeping claims about quantification would have the form of a mostly unsubstantiated thesis about the impossibility of quantification as such to express the important parts of a source concept such as intelligence. A further step in this direction would be to argue that not only any quantification but no explication in general can be successful in dealing with this concept.<sup>2</sup> However, it can become tricky to evaluate this kind of claims and the examples below should serve as a way to strengthen the reader's intuition about what it could mean to talk about limits of expressivity in this way.

Not only is there more to explication than quantification, there's also more to learn about the dynamics of meaning in these kinds of situations. Carnap's own case is a curious example. He developed the theory of explication in his book about the logical foundations of probability (Carnap 1971). The situation he was dealing with there was the reverse of the one we encountered with IQ. The mathematics of probability theory was well worked out. What was missing was a meaningful interpretation. It turns out that there is more than one competing interpretation of probability. Different meanings related to probability can be used to interpret probabilities.

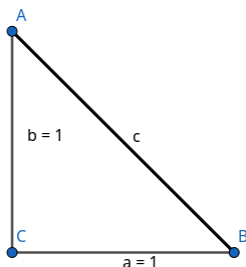
If, for example, we count past occurrences of throwing a dice and see that about one sixth of dice throws resulted in 6, we can stop here and say probability is nothing but this proportion in our counts (statistical interpretation of probability) or can interpret this proportion as meaning we now *believe* with a degree  $\frac{1}{6}$  that the next throw will result in 6 (degree of belief interpretation). Or we can say that the physical system of a dice throw has the propensity to

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2 A view that there can be no philosophical concept can be explicated in this sense is defended by Strawson (Strawson 1963). We leave the evaluation of his argument to the reader.

generate a sequence of results where the share of sixes will be  $\frac{1}{6}$  (objective/physical interpretation). Lastly there's also Carnap's own attempt at an interpretation. Logical probability would be a way to devise a way to see probability as a degree of logical support a premise has toward a conclusion. Ironically, this interpretation was the least successful of the ones we mentioned. But it would be too cumbersome to look at expressivity through the lens of the history of interpretation probability,<sup>3</sup> so we will look at a different selection of episodes in the history of expressivity.

Within mathematics, the story of the square root of two or length of the hypotenuse of the unit square is an example of what could be called an event in the history of expressivity. The Pythagoreans discovered a proof that the length of the hypotenuse cannot be expressed as a ratio of two natural numbers.



Diagonal of a unit square is not expressible as a ratio of whole numbers.

Anecdotally, the proof was received with antagonism by certain Pythagoreans whose metaphysics assumed ratios formed the only means and therefore limits of expressivity. So, all distances should be expressible with ratios of whole numbers. In this exam-

3 See Gillies' overview for the debates around the interpretation of probability (Gillies 2000).

ple we have a clear limit breaking event in the expressive power of one formal system and the possible extension to a new one, which would include a class of numbers the length of this diagonal. These are later named Real numbers, but it took mathematicians a long time to analyze this class as a mathematical object, although some sophisticated definitions arose quite early.<sup>4</sup>

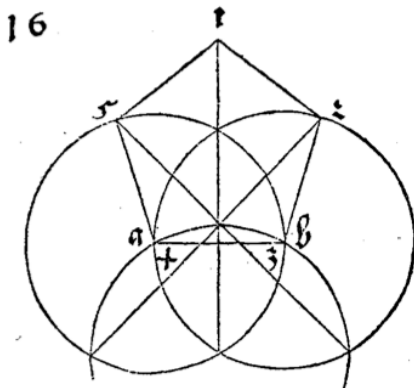
The point is that we have a single result about a single object that lies outside the bounds of a given formalism. If this object seems interesting enough to study, this then encourages us to adapt the formalism.

Another ancient example is the system of straightedge and compass constructions in euclidean geometry. Here we posit rules and moves that are allowed for constructing geometric objects. The expressivity of this system can then be analyzed in terms of constructibility of geometrical objects.

But rigorous proofs about what is and especially what isn't constructible were discovered only a long time after Euclid. Especially important here is the proof of the unconstructability of the regular heptagon. It is also clear that tweaking the rules will change the expressibility of our system. Famous examples are allowing the straightedge to rotate around a point – the so-called *neusis* method which facilitates a heptagon construction, and allowing to fold the paper as in the system of origami which enables the trisection of an angle (which is impossible in the case of euclidean constructions).

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4 Cf. Eudoxus of Cnidus.

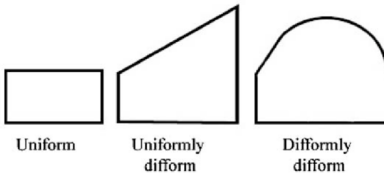


Dürer's attempt at a construction of a pentagon – unfortunately, it's off by less than 1%.<sup>5</sup> The classic construction by Euclid is much more complicated.

Another episode in expressivity or maybe even a whole arc is the development of the idea of acceleration. To oversimplify, in Aristotelian physics we posit only a uniform and a deformed motion of objects. An object like a planet can move uniformly in circles for example. A thrown stone can move uniformly for a while and then have its motion deformed by the ground when it falls. In this system it's hard to talk about various physical setups that have since become essential for physical ideas, for example pendulums and collisions.

The limits on expressivity by this concept of motion could be seen as a hindrance for the progress of physics. In the development of the theory of motion the medieval scholar Nicole Oresme thus had trouble expressing himself and so we find awkward statements about uniform motion opposed to the uniformly deformed and deformedly deformed motion (Kuhn and Hacking 2012).

5 For more on this see: <https://divisbyzero.com/2011/03/22/albrecht-durers-ruler-and-compass-constructions/>



Oresme's diagrams of different motion types reproduced by Babb (Babb 2005).

The question of acceleration was later tied to the debates about calculus, where another mathematical entity became the center of debates relating to expressivity. The object in question were the infinitesimals and in that episode many philosophers contributed to the debate as well. If complex numbers with their disparagingly named “imaginary” part present another example of a controversial expansion of the canonical system of numbers, the infinitesimals never made it into the mainstream (general education) set. Perhaps one reason for this is that the theory of calculus was formalizable without them – they are not a condition for all expressions of calculus. Despite this, we now have proofs about extending the class of numbers to such a degree that we admit infinitesimals as members of that class, it is called the hyper-real numbers, and with them we get a formalism for calculus (called “nonstandard analysis”) that is less cumbersome than the ordinary epsilon-delta maneuvers. One of the patterns to observe here is that a poorly understood formalism might also generate philosophical content.<sup>6</sup>

6 For a recent philosophical analysis of the debates regarding infinitesimals in relation to the formalization of hyperreal numbers see Hamkins' alternative history of math, which could have lead to the axiomatization of the continuum hypothesis (Hamkins 2024). Moreover, hyperreal numbers are not the only way to get to alternatives to classical calculus. Automatic (computer) differentiation systems can also rely on so called »dual numbers«, where much like the gesture of adding the imaginary unit ( $i^2=-1$ ), we instead add a dual number  $\epsilon^2=0$  and build the rules of calculus from there.



To get another perspective on expressivity we can look at another illuminating problem – the question of what it means to vote. On one hand, voting is of course a core concept for democracies and group decision making. On the other hand, it appears to be a simple counting problem. So in terms of the expressivity of our formal systems there almost shouldn't be any problems. The issue with voting is answering the question of *what* it is that we are trying to express as a counting problem. The drama that unfolds here is a kind of lack of content in the concept of voting that we are forced to fill whenever we're applying a formalism.

Let's start with a familiar design for a voting system (example from Bauer and Bračič 2022) and say that a vote is counted as adding 1 to one preferred party of the three parties seeking to win the election. Suppose there are five seats that we want to fill and that there were a thousand votes cast. 500 voted for party A, 300 for party B and 200 for party C. The first idea of how to represent the ratios of the votes into the ratio of the seats is proportionality. If we divide the number of voters with the seats, a seat should cost 200 votes. So party A gets 2 seats and B and C get 1 seat each. We notice that both A and B have 100 votes left, but that's not enough for a full seat.

Party	A	B	C	TOTAL
Votes	500	300	200	1000
Seats	?	?	?	5

Seats proportional	2	1	1	4+1 leftover
Seats D'Hondt method	3	1	1	5

The thing to notice here is that no analysis of the meaning of the concepts of voting or election will tell what the correct next step is. It is as if the formalism demands further determination of the concept

from outside the concept. In this situation we can treat this as a tie and flip a coin or invent another rule to break the tie. A common rule used for the next step in this situation is the D'Hondt method. Here we sort the quotients we get from dividing the parties' votes against increasing divisors and number of seats for each party is the greatest divisor in the sequence.

So in our example A gets the 1st seat because they have the most votes. B gets the second seat because 300 is more than 250. A got the 3rd seat because 250 ( $500/2$ ) is more than 200 (the number of votes C got). The fourth seat goes to party C, because 200 is more than  $500/3$  and  $300/2$ . And finally the fifth seat goes to A again, because  $500/3$  is more than either  $300/2$  or  $200/2$ . We could move forward in the same manner, but luckily there are no more seats, since this method could still generate a tie at some point. A reason to pick this and not some other tie breaking method is to reduce political fragmentation of elected parties. This is a pragmatic reason that election designers agreed is good for the workings of the elected government, but it is hardly an obvious conclusion derived from the meaning of voting itself.

Let's complicate the matter to observe another interesting result about expressivity. So suppose we extend the idea of what a vote is to a raking of the voter's preferences. When deciding between alternatives A, B and C, the vote is no longer just your top pick, but an ordering of the three. For example if you prefer A over B and B over C, you write A B C on the ballot and your vote will go in the first column of the table.

Let's now consider the results of this example election. The table below shows how many voters voted for each configuration, so 30 voters think A is better than B and B is better than C and so on. The question now is, how should we count these votes to determine the winner?

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#Votes	30	1	29	10	10	1
First choice	A	A	B	B	C	C
Second Choice	B	C	A	C	A	B
Third Choice	C	B	C	A	B	A

There's actually plenty of ways to do it but let's just take a look at these two. The first is called the Condorcet method. Here we compare all pairs and count the wins. So because A is above B in the 1st, 2nd and 5th column we count those votes as points for A against B, giving us 41. When we sum up the other columns where B is above A we get 40.

So we say A defeats B. When we do the same with A and C we also get an A win. And when we look at B and C we see that B wins. So A wins the tournament.

The second is Borda count. Here we turn the ranks of the votes into points, so that the third place gets 1 point, the second gets 2 and the first gets 3. Adding it all up gives us the result that B is the winner.

Counting method				Winner
Condorcet method	A vs B: 41/40	A vs C: 60/21	B vs C: 69/12	<b>A</b>
Borda count	A=182	B=190	C=114	<b>B</b>

We see that different counting methods give different winners. Again it's not immediately clear which one we should choose based on the meaning of voting. But there are practical reasons to talk about when choosing one of the existing methods of counting.

Borda count will favor moderate outcomes and reduce extreme polarization, while the Condorcet method will usually give the true majority preference. The downside to Borda count is that it enables strategic voting, while the Condorcet method can generate paradoxical cyclical situations with no winner. So is there a flawless way to count these kinds of votes? The answer is no. In fact

Arrow proved in 1948 that no method of counting would satisfy all of the criteria that he deemed are required for a good voting system (Arrow 1950).

Despite the seemingly negative result, his work was probably the first in social sciences to make use of this kind of axiomatic method and paved the way to a new way of thinking about optimal processes for collective decision-making and allocating scarce resources. Basically, it started the field of mechanism design. These kinds of problems go way beyond the scope of what is usually considered voting. Consider for example the engineering of decathlon ranking rules and Google's PageRank algorithm (Langville and Meyer 2012).

The obvious next story to consider is the unfolding of the concept of markets and auctions, which is another explication of a central social concept into what amounts to a crucial social technology. Let's say the story started when an economist (William Thomas Thornton) realized that sometimes the rules of an auction can give you different prices of a commodity. The classic examples are English and Dutch auctions (Mirowski 2017).

An English auction is the kind you see in movies. The auctioneer sets a price and people raise their hands in order to make a higher bid. This bidding continues until nobody makes a higher bid. These types of auctions are used to maximize revenue in competitive high interest and low supply markets. In a Dutch auction we start with a high price and lower it until a buyer decides to accept that price and the commodity is sold at that point. Dutch auctions are suited for high volume and time sensitive commodities. They both have certain drawbacks. English auctions are slower, but Dutch auctions don't have an unlimited maximum and can lead to lower prices.

For a more concrete example of market design, we can look briefly at the quite complicated European electricity market. In "day

ahead markets" suppliers bid their prices for every hour in the next day based on their production costs. The auctioneer then sorts these bids and determines the point where demand is met. Then the operator sets the price as the last highest price. The price of electricity is then set for all suppliers. This should incentivize low production cost suppliers because they get the difference between their production cost and the set price bonus. And because low production costs are associated with greener energy sources, this setup should incentivize green energy production. However, this idea can backfire since, for example, a sharp rise in gas prices can lead to the market rewarding coal burning plants for electricity production, which is what happened after the Russian invasion of Ukraine.

For comparison, the US electricity market also uses marginal pricing but the markets are fragmented on the state level. This means a lower integration of renewables. While in the EU it's normal that on a day with low wind production in Germany, they use French nuclear power instead. Other electricity markets (e.g., India or China) might not even have a marginal price system, but instead an average cost or a fixed rate. So again, it seems that the mere concept of markets is too vague to guide the design of markets by itself and heterogeneous considerations are required to guide the explication. Some rule must be used, but the choice depends on a range of factors and is reevaluated pragmatically.<sup>7</sup>

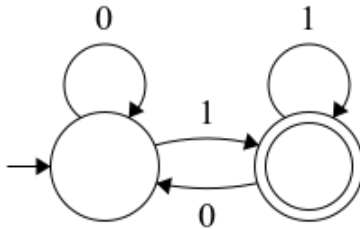
Returning now to a more formal setting: The theory of computation gives us what is perhaps the ideal example of what a theory of expressivity can look like. The way the theory is presented in its modern form (Sipser 2013) is as a sequence of formal types of

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7 Despite the fact these topics can be seen as problems of expression or explication, I'm not familiar with results in market design theory that would rely on expressivity or tackle the range of possible market mechanisms directly.

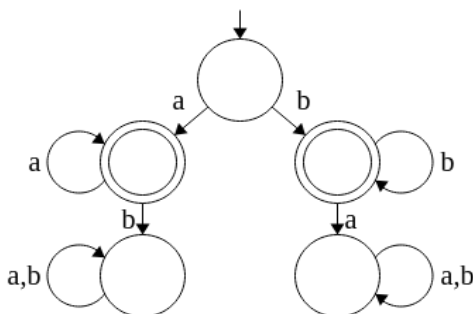
machines with increasing levels of expressivity. The expressive classes are determined by the kinds of strings - sequences of symbols – our type of machine can accept.

Conveniently corresponding to these classes of formal machines is a hierarchy of classes of generators called languages or grammars which produce the strings the machines on the corresponding level or expressivity can accept. So in this world the generators and recognizers align quite perfectly, unlike in the world of contemporary AI generation and recognition. We'll look at the three main levels starting with Finite state machines.



Example of a finite state automaton diagram that accepts all strings of ones and zeros ending with "1".

A finite state machine is a simple computer that can only track its current state and respond to a character in a sequence that we give it based on the state it's in. If at the end of the input the machine is in an accept state, we say that the machine accepted a string. The languages that correspond to finite states are called regular languages and they include strings that can contain simple patterns. For example all strings that start with an "a" and end with a "b". The moment we get something like a nested pattern of arbitrary length, like all strings that first have a number of As and then the same number of Bs, we're out of the realm of regular languages.



Example of a finite state automaton diagram that accepts strings with either only “a” or only “b” symbols.

Here the level of context dependent grammars begins and the strings they produce are recognized by machines called pushdown automata. Pushdown automata are similar to finite state machines except that here we add a simple memory called a stack. So when reading a string of letters one by one, the machine can also add something to the top of its memory or remember and remove the last element. This means it can't access any part of the memory without first removing all the parts that are on top of that element.

This kind of set up allows us to build the next level of expressive power that includes strings with simple nested patterns. An example for this is the previously mentioned equal parts As and equal parts Bs class of strings. Another example for using pushdown automata is checking if a string of parentheses is balanced the right way, meaning the string never closes a parenthesis it didn't open and it doesn't leave any open at the end of the string you're checking. These nested patterns cannot be expressed in finite

state machines but can in pushdown automata. However, if you add another element or layer to the nesting, you get out of the land of push down automata. For example, the rule what would check if there is an equal number of As, Bs and Cs in a string is unexpressible with pushdown automata. The same goes balancing two different types of parentheses.

For these kinds of patterns you need to build a Turing machine. Here, the picture changes a bit more. The previous machines worked on some input, but left it untouched even if they could write on a simple memory. Here the finite input comes on an infinite tape and a Turing machine can also write on this tape as well as move left or right as much as we like. This means it can access any point on the input or its memory at any time. With this setup a Turing machine can recognize extremely complex and nested patterns in strings. Basically it can recognize all the patterns any computer could if it had infinite memory.

One reason to talk about this hierarchy is that it's a clear example of systematically building up more expressive systems. This enables thinking about each class and the properties it has. Perhaps the most important result from thinking about the Turing machine is the proof that this expressive class becomes mathematically undecidable. This means that at this level you can pose problems that are unsolvable by any algorithm. In the previous two machines, we would know for sure that no matter which input we give them, they will make a decision whether to accept the input or not. But for Turing machines there are no such guarantees.

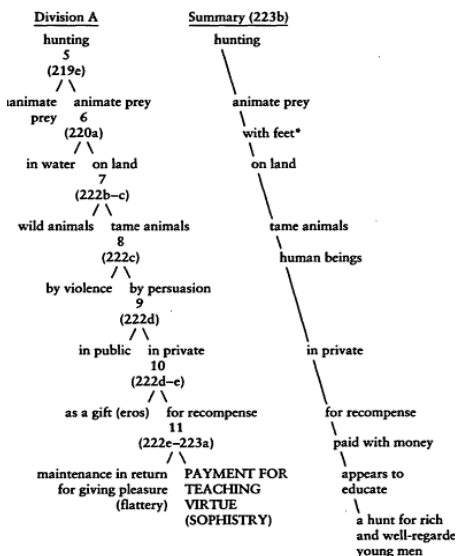
Unlike previous proofs about the limits of expressivity, this one seems bound to an expressive class itself, not some particular set of rules or axioms. Remember, with euclidean constructions we saw we can't make certain regular polygons with a straightedge and compass, but that really depends on the rules you set. But here,



the result seems to be that if you get to this expressive class by any means, you get undecidability. This was perhaps the clearest and most important episode in the history expressivity.

Let's look at one final and more speculative arc that can – if we squint enough – be seen in the history of philosophy and knowledge organization.

Philosophers sometimes think about concepts. Usually they theorize the structures of their relations between them or classifications of concepts. This is not so different from what in other contexts librarians and information scientists also have to think about. Let's compare some of the classic schemes of conceptuality. Take Plato's procedures in the *Sophist* as the first example (Plato and White 1993).



Dorter's diagram of Plato's *diaresis* (Dorter 1994).

This is a structure of relations between concepts that Plato one gets by diagramming a conversation procedure called diaresis. It seems that the motivation behind this technique is that if we complete this kind of structure we will know what there is to know about concepts and their relations or at least they will be sorted out in a way that people won't be confused about meaning anymore.

Number	Category	Example
1	Physical entity (material)	Man, horse
2	Quantity (size, weight)	Two-foot long
3	Nature (feature)	White, grammatical
4	Relationship (quantity)	Twice, half
5	Location (geography)	In Lucion, in the market
6	Time	Yesterday, a year ago
7	Posture	Lying and sitting
8	Status	Wearing shoes and armor
9	Action on	Cutting and burning
10	Be acted upon	To be cut and burned

Aristotle's categories as presented by Zhang and Tian (Li, Zhang, and Tian 2023).

With Aristotle we get the idea of categories. The categories here are abstracted from language. We get a classification, but it's no longer clear how the whole structure would look like. It probably won't be a simple diagram but a recursive application of the same categories – each object or sentence under consideration should be assessed by the categories. Here the idea that categories are somehow limiting the expressivity of possible thought becomes apparent.

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Table of Categories<sup>12</sup>

<p>1.</p> <p><b>Of Quantity</b></p> <p>Unity</p> <p>Plurality</p> <p>Totality</p>		
<p>2.</p> <p><b>Of Quality</b></p> <p>Reality</p> <p>Negation</p> <p>Limitation</p>		<p>3.</p> <p><b>Of Relation<sup>e</sup></b></p> <p>Of Inherence and Subsistence (<i>substantia et accidens</i>)</p> <p>Of Causality and Dependence (cause and effect)</p> <p>Of Community (reciprocity between agent and patient)</p>
	<p>4.</p> <p><b>Of Modality</b></p> <p>Possibility – Impossibility</p> <p>Existence – Non-existence</p> <p>Necessity – Contingency</p>	

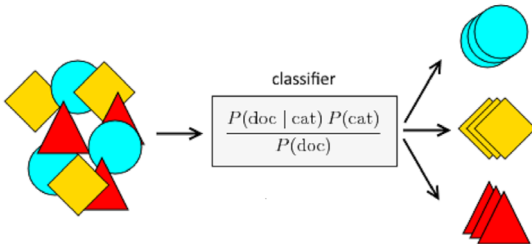
Kant's table of categories from the Critique of Pure Reason (Kant 2009).

Kant transformed the idea of categories into a more logical (not grammatical or merely predicative) framework. His table of categories works in a similar way, just that it is now modeled on his understanding of logic and simple logical relations instead of being empirically abstracted directly from natural language. It is at least at this point that it starts to become apparent that the logical structure of thought might not be the same as the grammatical structure of language (see also Russell 1905). But all of the examples so far are classical in the sense that the underlying idea of concepts is basically the same. They are closely related to sorting, classifying and logical connections.

This is also the reason why they can be linked to infrastructures such as libraries and typical problems of organization of knowledge. The pragmatic issues of library organizations are related to ideal conceptual structures. Practical issues of organization can lead

to specific failure modes in libraries that might have analogues in philosophical theories of conceptuality. It's easy to imagine that a book or other object that doesn't fit in any available category or that we might accidentally create categories that are too wide and risk becoming meaningless. Someone once said that it seems like the Slovenian library system the category "social novel" is at such a risk. But even in libraries we can already see that some books might fit into more than one category or have multiple categorical characteristics. Information science developed various tools to deal with these kinds of problems.

Interesting developments pop up with probabilistic methods in classification, such as the Bayes Classifier. Here, a relatively simple machine learning algorithm learns to classify objects based on the probabilities assigned to that object for belonging to a certain class. A classic example for this is the familiar spam filter, where the classifiers go through the email and try to decide if it is spam or not.



Bayes classifier diagram (Monthouel 2023).

On one hand this seems like an attempt to replicate categorical judgments, but it only takes a small leap in thinking to conceive of the probabilities themselves as representation of meaning. Infra-structurally this can appear as a meaningless content organization of an automated library or warehouse. It is meaningless in the sense

that there is no human-readable navigation that would serve as a system for shelving. But the point is that here space is used not as a shelf with a named category but a representation of optimal distances for objects that perform the function of a warehouse. The warehouse itself is then not to be understood as meaningless, but as a diagram of warehouse optimality which is itself a reflection of consumer behavior.

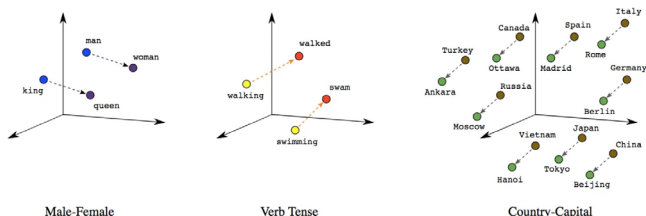


Illustration of 3D slices of vector embeddings taken from [google.developers](https://developers.google.com/earthengine/tutorials/vector-embeddings).

The last move in this direction is the formalization of conceptuality as vector embeddings that are a core component of current large language models. Here the words and their meanings are embedded as vectors in a very high-dimensional space; these dimensions are then thought to represent possible abstract relations. In the illustration above we take 3D slices of some of these many dimensions to show how the direction of the vectors can be thought of as a common abstract relation. The next trick is that this embedding can be used for other objects to express different multidimensional relations such as similarity of images or whole documents which allow us to express different modes of searching for and generating content such as generative models or vector databases of text content.

The vector embedding is the last formalism in this essay's sequence and so also gives us a direction of the expansion of possible ways of formalizing conceptuality. The embeddings themselves are hardly fully exhausted in terms of their expressivity, but it's unclear what sort of ideas are needed to make the most of it.

The question of metaexplication then is the question whether we can find a way to make correct or at least better explications based on such histories of expressivity. It would be a shame if a good thing is just one idea or a formal tweak away from being expressible or if there is a way to know if some of them are forever inaccessible. I hope that you find something useful in these episodes and in that expressivity state of mind.

## Sources

- Arrow, Kenneth J. 1950. "A Difficulty in the Concept of Social Welfare." *Journal of Political Economy* 58 (4): 328–46. <https://doi.org/10.1086/256963>.
- Babb, Jeff. 2005. "Mathematical Concepts and Proofs from Nicole Oresme: Using the History of Calculus to Teach Mathematics." *Science & Education* 14 (3–5): 443–56. <https://doi.org/10.1007/s11191-004-7937-y>.
- Bauer, Andrej, and Katja Bračič. 2022. "Volilni Sistemi Na Slovenskih Lokalnih Volitvah." Javno Predavnje. <https://volitve.fmf.uni-lj.si/javno-predavanje-2022.html>.
- Carnap, Rudolf. 1971. *Logical Foundations of Probability*. London: Univ. of Chicago Press.
- Dorter, Kenneth. 1994. *Form and Good in Plato's Eleatic Dialogues: The "Parmenides", "Theaetetus", "Sophist", and "Statesman"*. Berkeley, Calif.: University of California Press.
- Gillies, Donald. 2000. *Philosophical Theories of Probability*. Philosophical Issues in Science. London ; New York: Routledge.
- Hamkins, Joel David. 2024. "How the Continuum Hypothesis Could Have Been a Fundamental Axiom." *arXiv*. <https://doi.org/10.48550/arXiv.2407.02463>.
- Kant, Immanuel. 2009. *The Critique of Pure Reason*. 15th printing. The Cambridge Edition of the Works of Immanuel Kant. Cambridge: Cambridge University Press.
- Kuhn, Thomas Samuel, and Ian Hacking. 2012. *The Structure of Scientific Revolutions*. 4th ed. Chicago: University of Chicago press.
- Langville, Amy N., and Carl D. Meyer, eds. 2012. *Who's #1? The Science of Rating and Ranking*. Princeton: Princeton University Press. <https://doi.org/10.1515/9781400841677>.
- Li, Zongrong, Yiqiong Zhang, and Aijing Tian. 2023. "Comparison and Analysis of the Characteristics of Natural Information and Social Information—From Aristotle's '10 Material Category' to '12 Information Attribute' of Information Science." In *IS4SI Summit 2023*, 87. MDPI. <https://doi.org/10.3390/cmsf2023008087>.
- Mirowski, Philip. 2017. *The Knowledge We Have Lost in Information: The History of Information in Modern Economics*. Oxford: Oxford University Press.
- Monthouel, Alexandre. 2023. "Spam Classifier." 2023. [https://anOnImity.com/posts/spam\\_classifier/](https://anOnImity.com/posts/spam_classifier/).
- Plato, and Nicholas P. White. 1993. *Sophist*. Indianapolis: Hackett Pub. Co.
- Russell, Bertrand. 1905. "II.—ON DENOTING." *Mind* XIV (4): 479–93. <https://doi.org/10.1093/mind/XIV.4.479>.
- Sipser, Michael. 2013. *Introduction to the Theory of Computation*. Third edition, International edition. Australia Brazil Japan Korea Mexiko Singapore Spain United Kingdom United States: Cengage Learning.
- Strawson, Peter Frederick. 1963. "Carnap's Views on Constructed Systems versus Natural Languages in Analytic Philosophy." In *The Philosophy of Rudolf Carnap*, 503–18.

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Expressivity as a Condition of Expression



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